DEFINITION

Vector Form of a Line

Let ℓ be a line and let $\vec{d}$ and $\vec{p}$ be vectors. If $\ell = \{\vec{x} : \vec{x} = t\vec{d} + \vec{p} \text{ for some } t \in \mathbb{R}\}$, we say the vector equation

$$\vec{x} = t\vec{d} + \vec{p}$$

is ℓ expressed in ***vector form***. The vector $\vec{d}$ is called a ***direction vector*** for ℓ.

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11 Let $\ell \subseteq \mathbb{R}^2$ be the line with equation $2x + y = 3$, and let $L \subseteq \mathbb{R}^3$ be the line with equations $2x + y = 3$ and $z = y$.

11.1 Write ℓ in vector form. Is vector form of ℓ unique?

11.2 Write L in vector form.

11.3 Find another vector form for L where both "$\vec{d}$" and "$\vec{p}$" are different from before.

12 Let A, B, and C be given in vector form by

$$\overbrace{\vec{x} = t\begin{bmatrix}1\\2\\3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}}^{A} \qquad \overbrace{\vec{x} = t\begin{bmatrix}-1\\1\\1\end{bmatrix}+\begin{bmatrix}-1\\1\\2\end{bmatrix}}^{B} \qquad \overbrace{\vec{x} = t\begin{bmatrix}2\\-1\\1\end{bmatrix}+\begin{bmatrix}1\\1\\1\end{bmatrix}}^{C}.$$

12.1 Do the lines A and B intersect? Justify your conclusion.

12.2 Do the lines A and C intersect? Justify your conclusion.

12.3 Let $\vec{p} \neq \vec{q}$ and suppose X has vector form $\vec{x} = t\vec{d} + \vec{p}$ and Y has vector form $\vec{x} = t\vec{d} + \vec{q}$. Is it possible that X and Y intersect?